Strategy-Proof Mechanisms for Interval-Obnoxious Facility Location with Two Endpoints on a Line

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Abstract The placement of undesirable public projects—such as electromagnetic-emitting base stations, noisy public festivals, or disruptive construction zones—presents a fundamental challenge in social choice and urban planning. In many practical scenarios, the impact of such projects is not confined to a single discrete point but extends over a continuous region or a fixed time window. Citizens, likewise, have activity ranges rather than point locations, and they experience disutility proportional to the extent their activity areas overlap with the project. Because the facility is inherently obnoxious, citizens naturally prefer to minimize this overlap. Furthermore, since individuals hold private information about their spatial or temporal preferences, they may strategically misrepresent their activity domains to push the facility further away. Consequently, the central planner must design a social choice mechanism that not only optimizes aggregate or worst-case societal welfare but is also robust to strategic manipulation; in other words, the mechanism must be strategy-proof.

To formalize this problem, we introduce a novel facility location game in which both the facility and the agents' preferences are represented as intervals. Specifically, the planner must locate an obnoxious facility of a predetermined, fixed length k on the unit interval $[0, 1]$, with the two endpoints of the facility serving as the primary decision variables. Each agent i reports a private activity interval $[x_i, y_i] \subseteq [0, 1]$. An agent's cost is determined solely by the length of the intersection between her reported interval and the facility's interval, and her utility is inversely related to this overlap.

We design strategy-proof mechanisms to determine the location of an interval that optimize two objectives: maximizing total utility (social welfare) and maximizing the minimum utility. For each objective, we propose both deterministic and randomized mechanisms and establish their approximation guarantees relative to the social optimum.

Specifically, to maximize social welfare, we present a deterministic mechanism that achieves a $\frac{4}{3}$-approximation, as well as a randomized mechanism whose approximation ratio improves with k, expressed as $\frac{7-6k}{7-6k+2p(k-1)}$ for a parameter p. For maximizing the minimum utility, we provide a deterministic 2-approximation mechanism and a randomized $\frac{4}{3}$-approximation mechanism. All mechanisms are simple to describe and are strategy-proof. Our results extend the classical facility location game literature by introducing interval-based preferences and dual-point decisions, thereby contributing to the understanding of mechanism design under obnoxious preferences.